ISI – Bangalore Center – B Math - Physics I – Back Paper Exam
Date: 16 July 2015. Duration of Exam: 3 hours
Total marks: 50
Answer ALL Questions

Q 1 [Total Marks: 5]

An insect flies on a spiral trajectory such that its polar coordinates at time t are given by $r = be^{\Omega t}, \theta = \Omega t$ where b and Ω are positive constants.

Find the velocity and acceleration vectors at time t.

Show that the angle between these vectors is always $\pi/4$.

Q 2 [Total Marks: 2+3+5=10]

The equation of motion of a body projected vertically upwards with speed u in a medium that exerts a drag force $-mKv$ is given by
$m\frac{dv}{dt} = -mg - mKv$ where K is a positive constant and v is the instantaneous speed of the body.

2a.) Show that the body reaches a terminal speed and find the terminal speed.

2b.) Show that the time needed to reach the maximum height is $\frac{1}{K}\ln(1+\frac{Ku}{g})$.

2c.) Show that the maximum height reached is $\frac{u}{K} - \frac{g}{K^2}\ln(1+\frac{Ku}{g})$ [Hint: Integrate the expression for dz/dv, to find z as a function of v, and then substitute the appropriate value of v for the maximum height.]

Q 3 [Total Marks: 10]

A partially damped harmonic oscillator with natural frequency Ω and subjected to a damping force satisfies the equation:
$\frac{d^2x}{dt^2} + 2\kappa\frac{dx}{dt} + \Omega^2 x = 0$ where $\kappa = 0$ for $x < 0$ and $\kappa = K$ for $x \geq 0$ with constants $K > 0, \Omega > 0$.

3a.) If at $t = 0$, $x = -a$ with $a > 0$ and the initial speed is zero, draw the graph of x vs t for $K < \Omega$ and separately for $K > \Omega$.

3b.) Show that for $K < \Omega$ the time period for crossing the origin in the same direction is given by $\pi(\Omega + \Omega_D)/(\Omega\Omega_D)$ where $\Omega_D = (\Omega^2 - K^2)^{1/2}$

Q 4 [Total Marks: 10]

4a.) Show that in the absence of external forces, when two particles undergo a collision process, the particles stay in one and the same plane before and after the collision.

4b.) Show the when a moving particle collides elastically with another particle at rest with the same mass, EITHER the moving particle comes to a complete stop and the other particle moves with the same velocity as the first particle OR both particles move in such a way that their trajectories are orthogonal to each other.

Q 5 [Total Marks: 4+6=10]

5a.)

Show that for a rigid body of mass M rotating with angular speed ω around a fixed axis going through its CM which may be moving with velocity $\bar{V}$, the total Kinetic energy is the sum of the kinetic energy of the CM ($\frac{1}{2}MV^2$) plus the rotational energy ($\frac{1}{2}I\omega^2$))around the CM where I is the Moment of Inertia around the axis of rotation.
Will this hold true if ω is time dependent?

5b.)A uniform hollow cylinder of mass M and radius b is rolling without slipping down a rough plane inclined at an angle α to the horizon.

Show that the constraint forces do no work.

Apply the conservation of energy and show that the acceleration of the cylinder is $\frac{1}{2}g\sin\alpha$, [The relevant moment of inertia is Mb^2]

Q 6 [Total Marks: 5]

A particle of mass m is moving in a two dimensional central force field derivable from a potential given by $U(x, y) = \frac{1}{2}k(x^2 + y^2)$
Write the Lagrangian in the polar coordinates. Identify the cyclic coordinate and determine the associated conservation law.